Combining (3) and (4) into the Adiabatic Invariance Theorem, we arrive at an expression for the frequency pulling:

$$\frac{\delta f}{f} = \frac{a^3 \kappa}{4b^2 h \rho_{01} J_1^2(\rho_{0m})} \tag{5}$$

where h is the height of the cavity and ρ_{0m} is the mth root of J_0, corresponding to the TM_{0m0} mode in the cavity, and where h is 2.25 inches.

The first-order perturbation relation for dielectric constant in terms of frequency shift[1,3] is

$$\kappa - 1 = 2J_1^2(\rho_{0m}) \frac{b^2}{a^2}\left(\frac{f_e}{f_s} - 1\right), \tag{6}$$

where f_s is the resonant frequency with the sample inserted, and f_e is the resonant frequency of the empty cavity. The error in $(\kappa - 1)$ due to errors in f_s and f_e is easily obtained from (6), and with the expression for the error in frequency, (5), this error in dielectric constant is finally:

$$\frac{\delta(\kappa - 1)}{\kappa - 1} = -\frac{a}{2\rho_{01} h}\frac{f_e}{f_s} \approx -\frac{1}{2\rho_{01}}\frac{a}{h}. \tag{7}$$

Results and Discussion

A series of direct measurements was made on materials of relative dielectric constants between 1 and 10. Ratios of a/h up to 0.22 were investigated. In no case did the value of $m^2 a^2 \kappa / b^2$ exceed 0.11.

If the experimental values of

$$\frac{\delta(\kappa - 1)}{(\kappa - 1)} \Big/ \frac{a}{h}$$

are plotted as a function of κ (Fig. 2), (7) predicts that the curve will be a constant function with value equal to $1/2\rho_{01} = 0.208$, independent of κ and of the radial index of the mode of oscillation within the TM_{0m0} family. Experimentally, this value was found to be 0.21 for the TM_{010} mode and 0.2 for the TM_{020} mode.

The experimental behavior of the normalized frequency shift was next investigated. If a log-log plot is made of $\delta f/f$ against κ, as in Fig. 3, a dependence on some power of κ higher than unity is indicated, *i.e.*, $\kappa^{1+\Delta}$. From the slope, Δ was found to be about 0.16. The approximate theory developed herein accounts only for $\Delta = 0$. However, a strict first-order perturbation theory[7] indicates that Δ is greater than zero.

Therefore, we shall empirically modify (5) to

$$\frac{\delta f}{f} = K_m \frac{a^3 \kappa^{1+\Delta}}{b^2 h}, \tag{8}$$

where $\Delta = 0.16$ and K_m is a function only of m, the radial mode index. The theory (5) gives $K_1 = 0.386$ and $K_2 = 0.897$. Experimentally the values of K_1 and K_2 are found to be 0.32 and 0.87, respectively. Fig. 4 shows the experimental data that determined K_m.

It is interesting to note that although the simple equation (5) departs significantly from the experimental equation (8), this causes no discernible error in (7) which depends on (5). [It may be seen that (7) is well supported by the experimental data in Fig. 2.] This has not been investigated in detail, but it would seem that (6) also departs from experiment, and that when (5) and (6) are combined to obtain (7) there is a cancelling of errors.

Acknowledgment

The authors are indebted to Dr. George Birnbaum for his early leadership in problems of cavity dielectric measurements, out of which this investigation arose.

[7] D. M. Kerns and H. E. Bussey, manuscript in preparation.

Correction

D. S. Lerner and H. A. Wheeler, authors of "Measurement of Bandwidth of Microwave Resonator by Phase Shift of Signal Modulation," which appeared on pages 343–345 of the May, 1960, issue of these Transactions, have brought the following to the attention of the *Editor*.

Reference [8], which appears on page 345, should read:

F. H. James, "A method for the measurement of high Q-factors," *Proc. IEE*, vol. 106, pt. B, pp. 489–492; September, 1959. (Recent proposal of the subject method.)